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INTERACTION OF RADIO WAVES IN THE IONOSPHERE

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ABSTRACT

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A series of formulae is derived to demonstrate that the interaction in the ionosphere between a strong amplitude-modulated and a weak nonmodulated wave affect not only the absorption but also the dielectric constant, as a result of which the weak wave becomes not only amplitude-modulated but also phase-modulated.

The formulae are considered sufficiently accurate when transmitter power does not exceed 100 to 200 kw. Although investigations made both during the day and at night are in agreement with the computed data, it is stressed that the calculations should be considered as having only a general indicative value.

Author

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1. Usually, when examining the interaction of two radio waves in the ionosphere — a weak nonmodulated and a strong amplitude modulated by sound frequency — it is customary to limit oneself to the consideration of amplitude modulation imparted by the "strong" wave upon the "weak" wave (cross modulation). As is well known, the latter is conditioned by the fact that the effective

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* K voprosu o vzaimodeystvii radiovoln v ionosfere.

number of collisions varies under the effect of the "strong" wave, and by the strength of that the absorption coefficient of the "weak" wave changes also. However, generally speaking the dielectric constant depends on the effective number of collisions also. By the strength of that it is obvious that the weak wave will not only be amplitude-modulated but also phase-modulated.

Let us consider this question at further length.

Assume that an amplitude-modulated wave propagates in the ionosphere and that its field at the lower boundary of the ionosphere has the form

$$E_1 = E_{10}[1 + M\cos(\Omega t)]\cos(\omega_1 t). \qquad (1)$$

The effective number of collisions having varied under the effect of this wave may be written in the form:

$$\nu_{э\phi\phi} = \nu_0 + \Delta\nu_0 + \Delta\nu_\Omega + \Delta\nu_{\omega_1}. \qquad (2)$$

Here ν_0 is the value of ν_{eff} at $E_1 = 0$, $\Delta\nu_0$ is the correction for ν_0 containing the periodical terms of the sonic frequency Ω, 2Ω , $\Delta\nu_{\omega_1}$ is the correction containing the periodical terms of the high frequency $2\omega_1$.

Let us consider that the field E_1 is not very great so that we may limit ourselves to accounting for the nonlinearity in the first approximation. Besides, let us neglect the term $\Delta\nu_0$ (for in the considered approximation $\Delta\nu_0 \ll \nu_0$). and also the usually small term $\Delta\nu_{\omega_1}$. For the same reason we shall not consider the appearing waves with combined bearing frequencies [1]. We shall limit ourselves within the framework of "elementary" theory *. Finally, we

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* It is well known that provided one is only interested in the variations along the sonic frequency, there is no necessity of solving the nonlinear wave equation: It is sufficient to confine oneself to the "quasistationary" solution of the linear equation. That is what we are expected to do in the following.

shall consider that $\omega_1 \ll \omega_H$, (ω_H being the gyromagnetic frequency). Then for $\Delta\nu_2$ we shall have : [2] $\Delta\nu_2 = M_1\cos(\Omega t - \varphi_1) + M_2\cos(2\Omega t - \varphi_2)$;

$$M_1 = \frac{e^2E_{10}^2\nu_0^2 M\cos^2\beta}{m^2v^2(\omega_1^2+\nu_0^2)\sqrt{\Omega^2+(\delta\nu_0)^2}}; \qquad \mathrm{tg}\,\varphi_1 = \frac{\Omega}{\delta\nu_0}; \tag{3}$$

$$M_2 = \frac{e^2E_{10}^2\nu_0^2 M^2\cos^2\beta}{4m^2v^2(\omega_1^2+\nu_0^2)\sqrt{4\Omega^2+(\delta\nu_0)^2}}; \qquad \mathrm{tg}\,\varphi_2 = \frac{2\Omega}{\delta\nu_0},$$

where the following designations were adopted : e, m are respectively the charge and the mass of the electron, v is its mean motion velocity, δ is the mean dose of energy lost be the electron at collision with heavy particles (molecules or ions), β is the angle between E_1 and H_o (H_o being the intensity of the Earth's magnetic field).

The "weak" wave $E_2 = E_{20}\cos(\omega_2 t - \varphi)$, traversing the disturbed part of the ionosphere will result amplitude-modulated and, as pointed out earlier — phase-modulated. Let us consider the wave phase in a more detailed fashion. For simplicity we shall assume that the propagation of the wave takes place in a homogenous medium (when dealing with numerical estimates below, we shall make more precise the ionosphere model). We may then write for the phase φ :

$$\varphi = \frac{\omega_2}{c} n(\omega_2)\, z, \tag{4}$$

where $n(\omega_2)$ is the wave's/E2 refractive index, c is the speed of light, z — the wave's path.

Under the condition which is usually satisfied (except for the reflection region), we have :

$$n(\omega_2) = \sqrt{\varepsilon(\omega_2)} = \sqrt{1 - \frac{a}{\omega_2^2 + \nu^2_{\phi\phi}}}, \tag{5}$$

where $a = 4\pi e^2 N/m$, N is the electron concentration. Substituting $\nu_{eff} = \nu_0 + \Delta\nu_\Omega$ and limiting ourselves to accounting the terms containing $\Delta\nu_\Omega$ in the first power, we shall obtain after rather simple transformations :

$$n(\omega_2) \simeq n_0 + \frac{a\,\nu_0\,\Delta\nu_\Omega}{(\omega_2^2 + \nu_0^2)^2 n_0}; \qquad n_0 = \sqrt{1 - \frac{a}{\omega_2^2 + \nu_0^2}}. \tag{6}$$

Then, according to (4), we shall have for the wave phase:

$$\varphi = \frac{\omega_2}{c} n_0 z + \Delta\varphi, \tag{7}$$

and after substituting all the quantities, we shall obtain the following expression for $\Delta\varphi$:

$$\Delta\varphi = \frac{\omega_2}{c} z \frac{a\,\nu_0\,\Delta\nu_\Omega}{(\omega_2^2 + \nu_0^2)^2 n_0} = \beta_\Omega \cos(\Omega t - \varphi_1) + \beta_{2\Omega} \cos(2\Omega t - \varphi_2);$$
$$\beta_\Omega = \frac{\omega_2}{c} z \frac{4\pi e^4 N \nu_0^3 E_{10}^2 M \cos^2\beta}{m^3 v^2 (\omega_1^2 + \nu_0^2)(\omega_2^2 + \nu_0^2)^2 \sqrt{\Omega^2 + (\delta\nu_0)^2}\, n_0};$$
$$\beta_{2\Omega} = \frac{\omega_2}{c} z \frac{\pi e^4 N \nu_0^3 E_{10}^2 M^2 \cos^2\beta}{m^3 v^2 (\omega_1^2 + \nu_0^2)(\omega_2^2 + \nu_0^2)^2 \sqrt{4\Omega^2 + (\delta\nu_0)^2}\, n_0}. \tag{8}$$

Thus, at the receiving spot, the ω_2 frequency wave will result not only amplitude-modulated but also phase-modulated with modulation frequencies Ω, 2Ω. The indices of phase modulations β_Ω and $\beta_{2\Omega}$ are determined by the expression (8).

$|\varepsilon(\omega)| \gg 4\pi\sigma(\omega)/\omega$

Let us make some remarks concerning those admissions which were made earlier, when deriving the expressions (8).

The fact that by assuming $\Delta\nu \ll \nu_0$, we limited ourselves to accounting only the terms containing $\Delta\nu$ in the power not above the first, imposes limitations to field intensity of the "perturbing" transmitter E_{10}. We may consider that formulae (8) are valid with a sufficient precision, while the power of this transmitter does not exceed 100 ÷ 200 kw (see for example [3]).

Further, when considering this we utilized formulae of elementary theory. It has been shown in a series of works (see for example [1, 2], that if we assume $v^2 = \bar{v}^2 = 8\,kT/\pi m$ or $v^2 = \bar{v}^2 = 3\,kT/m$, in (3) and consequently in (8), these formulae will be distinct from the corresponding kinetic theory formulae (at least for the case of electron collisions with molecules) by only the numerical factor close to the unity. In our case this is immaterial.

Finally, concerning the assumption about the homogeneity of the medium. If we account for the inhomogeneity of the medium, we shall obviously obtain in the geometrical optics approximation instead of (8) the following expression :*

$$\beta_2 = \frac{\omega_2}{c}\frac{4\pi e^4 M}{m^3}\int\limits_s \frac{N\nu_0^3 E_{10}^2\cos^2\beta}{v^2(\omega_1^2+\nu_0^2)(\omega_2^2+\nu_0^2)^2\sqrt{\Omega^2+(\delta\nu_0)^2}\,n_0}\,ds;$$

$$\beta_{22} = \frac{\omega_2}{c}\frac{\pi e^4 M^2}{m^3}\int\limits_s \frac{N\nu_0^3 E_{10}^2\cos^2\beta}{v^2(\omega_1^2+\nu_0^2)(\omega_2^2+\nu_0^2)^2\sqrt{4\Omega^2+(\delta\nu_0)^2}\,n_0}\,ds, \qquad (8a)$$

where s is the wave's E_2 path in the region of the ionosphere "perturbed" by the effect of the wave E_1. These are the formulae we shall utilize below when conducting concrete calculations.

2. It is interesting to compare the magnitude of the phase modulation with the depth of amplitude modulation indiced upon the wave E_2 by waves of the interefering station E_1. In the considered approximation we shall have for the depth of cross amplitude modulation :

$$M_2 = \frac{e^2 E_{10}^2 \nu_0 M\cos^2\beta\int \varkappa_0 ds}{m^2 v^2(\omega_1^2+\nu_0^2)\sqrt{\Omega^2+(\delta\nu_0)^2}}.$$

* this is quite sufficient whene there are no sharp inhomogeneities in the medium.

In case of a uniform medium $\int \varkappa_0 \, ds$ is replaced by $\varkappa_0 z$. Further, for the amplitude absorption coefficient $\varkappa_0$ we shall utilize the expression [2] :

$$\varkappa_0(\omega_2) = \frac{2\pi\sigma(\omega_2)}{cn_0(\omega_2)},$$

where $\sigma(\omega_2) = e^2 N \nu_0 / m(\omega_2^2 + \nu_0^2)$ is the ionosphere conductivity. We shall then obtain for M_2:

$$M_\Omega \simeq \frac{z}{c} \frac{2\pi e^4 N \nu_0^2 E_{10}^2 M \cos^2\beta}{m^3 v^2(\omega_1^2 + \nu_0^2)(\omega_2^2 + \nu_0^2)\sqrt{\Omega^2 + (\delta\nu_0)^2}\, n_0}. \tag{9}$$

From (8) and (9) we shall find

$$\frac{\beta_\Omega}{M_\Omega} \simeq \frac{2\omega_2\nu_0}{\omega_2^2 + \nu_0^2}. \tag{10}$$

In the case of a nonuniform medium the correlation (10) remains obviously valid, if we view ν_0 as a certain mean value of this quantity in the interaction region.

It is clear from (10) that if we measure the index of phase modulation simultaneously with the measurement of the depth of the parasitic amplitude modulation, it is easy to determine the quantity ν_0. Further, studying the dependence of M_Ω or β_Ω on the modulation frequency Ω, we may determine, as is well known the quantity $\delta\nu_0$ experimentally. Consequently it will be possible to determine from these experiments the value of the parameter δ directly under the ionosphere conditions and with a sufficient precision.

Let us clarify as to whether the index of parasitic phase modulation is sufficiently great for direct measurements.

It may already be seen from formula (10) that under the condition $\omega_2 \sim \nu_0$ the quantities β_Ω and M_Ω will be of the same order. Let us note that phasometric devices may have a sufficient sensitivity (see for example [4]). That is why the possibility of experimental study of parasitic phase modulation leaves no doubt.

More detailed calculations were carried out for the expected values β_Ω and M_Ω in two cases.

a) Night Conditions. The following layer model is adopted: the layer begins at 70 km altitude. From 70 to 80 km $N = N_0 = 100$ electron cm^{-3}, as of 80 km the variation of N takes place according to linear law, while at 90 km N reaches the value $N = 2.5 \cdot 10^3$ el cm^{-3} As to ν_0, it is taken for granted that it varies according to exponential law from the value $\nu_0 = 10^7 \sec^{-1}$ at $z = 70$ km, to $\nu_0 = 8 \cdot 10^5 \sec^{-1}$ for $z = 90$ km. Further it is considered that the power of the "interfering" transmitter is 100 kw, and its frequency $f_1 = \omega_1/2\pi = 200$ kc/s, the antenna directed action coefficient is the unity. The frequency of the "modulable" transmitter $f_2 = \omega_2/2\pi = 400$ kc/s, the distance between it and the reception point is 250 km, while the "interfering" transmitter is situated half-way between the "modulating" transmitter and the reception point.

The path of the wave ω_2 was approximated in the form of lateral sides of an isosceles triangle. To effect the numerical integration in formila (8a), the ionosphere was broken up into layers of 2 km width, the parameter values for each layer being taken equal to their value in the median part. It was then obtained that $\beta_\Omega \simeq 9.0 \cdot 10^{-2}$. A similar computation of M gave the following result : $M_\Omega - 8.7 \cdot 10^{-2}$, which is close to the values observed in concrete conditions.

b) Daytime Conditions. The following model of the lower ionosphere has been adopted: from 55 to 70 km N varies according to linear law from $N = 0$ to $N_{70} = 10^3$ electron cm^{-3}. Further, to 80 km we considered $N =$ const (higher layers are of no interest to us, since the wave ω_2 is reflected below $z = 80$ km). The remaining parameters are adopted the same as in the case a).

The computations gave the following results :

It should be noted that if one observes and measures the cross amplitude modulation, the depth of which is less than 0.5%, which in our opinion is quite difficult, the measurement of the parasitic phase modulation index of the order of $10^{-3} \div 10^{-4}$ is quite possible.

Obviously, the above numerical computations must only be considered as tentative. They however unquestionably attest to the fact that the study of the parasitic phase modulation may significantly complement those informations about the lower ionosphere which are obtained at the analysis of data on cross modulation. Particularly important is the circumstance that the parasitic phase modulation may apparently be measured in daytime conditions: the corresponding experimental data would provide valuable informations about the D-layer, which at present is still little studied.

Let us note in conclusion that we also made an attempt to study the parasitic phase modulation appearing on account of ionosphere nonlinearity (self-action). This attempt failed, because the transmitter itself had a parasitic phase modulation that could not have been separated from the phase modulation occurring in the ionosphere.

In investigating radio wave interaction no such difficulties are to be expected, since the received wave ω_2 does not modulate in the transmitter, while the small parasitic phase modulation of the "perturbing" wave ω_1 will practically have no effect in the final result.

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****** E N D ******

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